Barth

A NEW METHOD OF MEASURING SHORT

HALF-LIVES.

A Study of Reasoning about Self-Alive

Abstract of

A Thesis

Presented in Partial Fulfillment of the Requirements for

the Degree Master of Science

By

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The Ohio State University

1950

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Advisor

A NEW METHOD OF MEASURING SHORT HALF-LIVES

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A review of several methods of measuring the half-lives of short-lived radioisotopes is presented. Then a new method is described which utilizes a cathode ray oscilloscope in conjunction with three phototubes and associated electrical circuits. The design and construction of the concomitant apparatus is taken up in detail. The method described is then applied to the specific problems of measuring the half-lives of the isotopes aluminum twenty-five and aluminum twenty-six, which result from the bombardment of magnesium twenty-four and magnesium twenty-five respectively with protons accelerated by a Van de Graaff generator. The values obtained for the half-lives are reported as follows: aluminum twenty-five -- seven and eleven hundredths seconds plus or minus thirteen hundredths of a second; aluminum twenty-six -- seven and sixty-one hundredths seconds plus or minus twenty-one hundredths of a second. In conclusion the capabilities and limitations of the new method described are delineated, and future applications are suggested.

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Acknowledgement

The author wishes to take this opportunity to express his sincere appreciation to Professor James C. Harris and Professor John M. Cooper for their constant assistance and guidance throughout this work. Thanks are also due to Donald A. Green for his assistance in taking the data.

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Historical Review and Introduction

Before proceeding to a description of a new method of measuring the half-lives of short-lived radioisotopes it is perhaps advisable to describe some of the methods which have been used successfully in the past. In this regard Rowlands[1] has given an excellent review of several methods, some of which will now be discussed.

Attempts to measure the half-life of Ra C' (1.55×10^{-4} sec.) gave rise to three different methods of measuring short half-lives, all of which depend on the radioactive daughter isotope of interest being formed from a radioactive parent whose half-life is long compared to that of the daughter. These methods may be called (1) the pulse-lengthening method, (2) the delayed pulse method, and (3) the oscilloscope method.

The pulse-lengthening method has been developed by Dunworth[2] and Rotblat.[3], [4] It utilizes two counters, one of which is sensitive to the beta radiation from Ra C and one of which is sensitive to the alpha radiation from Ra C'. The pulses from the two counters are amplified separately, made of uniform size, and then fed into a coincidence circuit. The length of the beta pulses is varied and the number of coincidences per unit time observed as a function of the length of the pulse. That the graph of the coincidences per unit time versus the length of the pulse is the inverse of

the Ra C' decay curve may be seen from the considerations below.

The general law of radioactive decay may be written as

$$N = N_o e^{-\lambda t} \qquad (1)$$

where N = the number of atoms of a radioactive material remaining at the end of a time t ; N_o = the initial number of atoms; and λ = the decay constant.

If N_o be put equal to one and the differential of (1) taken, the result is

$$dN = -\lambda e^{-\lambda t} dt \qquad (2) .$$

From (2) it may be seen that the probability for a given atom to live a time t and then decay in the interval from t to t+dt is $\lambda e^{-\lambda t} dt$. Thus the probability for the emission of an alpha particle during the time τ, or what is the same thing, the rate of coincidence when the beta pulse has a length τ is

$$C = N_o \int_o^\tau \lambda e^{-\lambda t} dt \qquad (3)$$

where λ is the decay constant of Ra C' and N_o is the initial number of Ra C' atoms.

Evaluating the integral in (3) leads to

$$C = C_{max} (1-e^{-\lambda \tau}) \qquad (4)$$

where C_{max} is the maximum coincidence rate, which will be attained when τ becomes infinite. For infinite τ, $C = C_{max} = N_o$. From (4) it can be seen that the coincidence rate increases exponentially with the beta-pulse length in a manner

determined by the decay constant λ. Hence it is possible to compute the half-life of Ra C' from the experimental relationship observed between the coincidence rate and the length of the beta pulse.

The delayed pulse method has been developed by Jacobsen and Sigurgeirsson [5]. As in the previous method two counters are used, one sensitive only to beta radiation and the other sensitive only to alpha radiation. The outputs of the two counters are fed into a coincidence circuit as before. Now, however, instead of lengthening the beta pulse the arrival of the beta pulse is delayed a variable length of time. As before the probability that an atom decays in the interval between t and t+dt is $\lambda e^{-\lambda t}\, dt$. When the pulse widths are short compared to the half-life being measured this is equivalent to the probability that a coincidence be recorded after the beta pulse has been delayed t units of time. Therefore the coincidence rate may now be expressed as

$$C = N_o \int_{\tau}^{\tau+\tau_c} \lambda e^{-\lambda t} dt \qquad (5)$$

where τ is the delay time of the beta pulse; τ_c is the maximum length of time by which two pulses may be separated and still be recorded as a coincidence; λ is the decay constant of Ra C' ; and N_o is the initial number of Ra C' atoms. Integration of (5) gives

$$C = N_o e^{-\lambda\tau} (1-e^{-\lambda\tau_c}) \qquad (6) .$$

If one now denotes by C_o the value of C when τ = o, it is

easily seen that

$$N_o = C_o/(1 - e^{-\lambda\tau_c}) \qquad (7) .$$

With this substitution (6) becomes

$$C = C_o\, e^{-\lambda\tau} \qquad (8) .$$

From (8) it is clear that a graph of the observed rate of coincidence versus the delay time of the beta pulses will give a decay curve for Ra C' from which the half-life may be easily computed.

The oscilloscope method has been developed by Rowlands.(6),(7) Two counters are used as in the previous two methods. This method does not, however, utilise a coincidence circuit. Instead, the beta pulse is used to trigger a single sweep on a cathode ray oscillograph and the alpha pulse is applied to the vertical deflection plates. By this means it is possible to observe directly the time intervals between pulses simply by noting the position of the alpha pulse on the oscillograph screen. The number of alpha pulses observed after a time delay τ will be

$$N = N_o \int_{\tau}^{\tau+\Delta\tau} \lambda e^{-\lambda t}\, dt \qquad (9)$$

where $\Delta\tau$ is the smallest unit of time observable on the oscillograph time scale being used; λ is the decay constant of Ra C' and N_o is the initial number of Ra C' atoms. As in the delayed pulse method this may be readily integrated to give

$$N = N_o'\, e^{-\lambda\tau} \qquad (10)$$

where now N_o' is the number of alpha pulses having a zero time

delay; by analogy to (7) N_0' is equal to N_0 ($1-e^{-\lambda\Delta\tau}$), so it is evident that a plot of alpha pulses versus observed delay times will give a decay curve for Ra C' from which the half-life can be easily determined.

It appears on the surface as though none of the three methods just described could be utilized to measure the half-life of a radioactive isotope which is the sole product nuclei of, say, a nuclear reaction and which decays to a stable isotope. Actually, however, De Benedetti and McGowan[26] have shown that the delayed pulse method may be used to measure the half-life of a single isolated radioactive material. Several other methods also applicable to this case will now be discussed.

In a method originated by Moseley and Fajans[9] and developed by Becher and Baerttner[9] a target is deposited on a disk which rotates at high speed. The target is then bombarded with nuclear projectiles from some sort of an accelerator, and the activity is measured as a function of the angular displacement from the point of bombardment. From this data the half-life of the activity may easily be computed.

Cassels and Latham[10] have used a cyclotron modulated to give short bursts of particles. In between the bursts, the counter measuring the radioactivity of the target is connected in turn for equal time intervals to each of ten

separate recorders. The half-life of the activity may then be easily computed from the data obtained. Jaeger and Latham claim that this method is good for half-lives ranging from 100 microsec. to 1 sec. The factor which sets a lower limit to the half-life measurable by this technique is the recovery time of the counter being used to measure the activity. The use of scintillation counters, which have recovery times on the order of 10^{-8} sec., would increase the range of applicability of this method considerably.

For half-lives longer than a few seconds, a target may be bombarded and then removed to an adjacent counter of some sort which measures the activity and the decay of the induced radioactivity. The main difficulties in this procedure are getting the radioactive target from the place of bombardment to the counter before the activity has decayed too much, and devising a timing mechanism to accurately fix the time intervals during which the counter counts. The number of counts obtained must also be recorded in some fashion. The counting interval should, of course, be reasonably small compared to the half-life being measured. Different research workers[1],[11],[12] have devised various methods to surmount the difficulties mentioned. For more specific information the literature may be consulted.

In general the methods for measuring short half-lives just described have limited ranges of applicability and require complicated electronic circuits which are not avail-

able commercially. The new method which will be described in this thesis has a range of applicability from approximately 1 millisec. to 1 min. and utilizes for the most part commercially available equipment. The associated electrical circuits which must be constructed are simple in design and not difficult to assemble. Although the method was designed for use with a Van de Graaff generator, it could easily be adapted for use with any other type of accelerator. After the general theory and construction of the equipment have been discussed, descriptions of two experiments utilizing the method will be given. The results of these experiments will be discussed and finally future applications of the method will be suggested.

General Description of Method and Equipment

The method discussed below was devised for use with the Van de Graaff generator at Ohio State University. A general description of the pieces of equipment necessary and the tasks performed by each follows.

A shutter was constructed to interrupt the Van de Graaff generator beam. Simultaneously with interrupting the beam the shutter actuates a microswitch which in turn actuates a relay raising a lead shield from between the target being bombarded and an active Geiger tube. The pulses from the

Geiger tube are fed into the vertical deflection plates of a cathode ray oscilloscope. The microswitch mentioned before also closes an electric circuit which triggers a single sweep of the electron beam across the face of the oscilloscope. Positioned at definite intervals along the time axis of the oscilloscope are three lucite tubes which pipe light to three separate type 931-A photomultiplier tubes. Each photomultiplier tube is followed by its own preamplifier and its own independent scaler. Thus during a single sweep of the electron beam across the face of the oscilloscope a certain number of pulses will be "seen" by each phototube and recorded on the appropriate scaler. This will give three points on a decay curve of the activity being counted. Of course, this routine must be gone through many times to obtain sufficient counts on each scaler to give a small probable error. The time base of the oscilloscope sweep must be adjusted to a value commensurate with the half-life of the activity being measured.

It is seen from the preceding description of the new method that the equipment necessary is:

(1) A Van de Graaff generator.

(2) A shutter for interrupting the Van de Graaff generator beam.

(3) A Geiger tube.

(4) A cathode ray oscilloscope.

(5) Three phototubes with individual preampli-

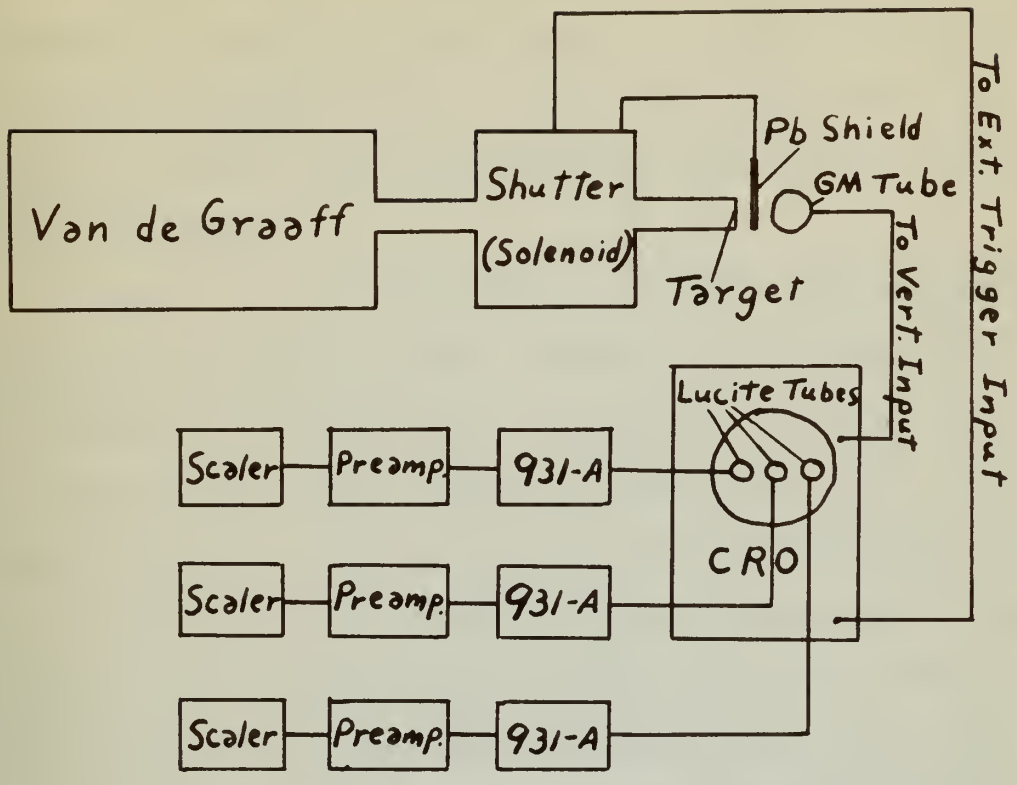

Figure 1. Sketch of arrangement of various equipment.

fiers and independent outputs.

(6) Three scalers.

(7) Various voltage supplies.

Figure 1 is a sketch illustrating the proper relationship of this equipment. Items (2) and (5) in the above list require a special description since they had to be designed and constructed.

Figure 2 is a sketch of the shutter used in this work. As can be inferred from the sketch the motivating force for the shutter is provided by a solenoid. When this solenoid is actuated by a hand switch in series with a 200 volt D. C. power supply the soft iron core is drawn inside the solenoid, cutting off the Van de Graaff beam and actuating the micro-switch. The solenoid draws 70 milliamperes of current. When the hand switch is turned off a spring draws the shutter back so that the Van de Graaff beam is again bombarding the target.

Figure 3 is a circuit diagram of one phototube and its preamplifier. There are two more identical units. Several general references were consulted in the design of these circuits. (13),(14),(15),(16),(17)

Figure 4 is a photograph of all three units complete, showing the lucite tubes and the framework by which the whole is fastened to the face of the oscilloscope. The lucite tubes were first shaped and then cemented to the 931-A phototubes by means of Canada balsam. The phototubes were

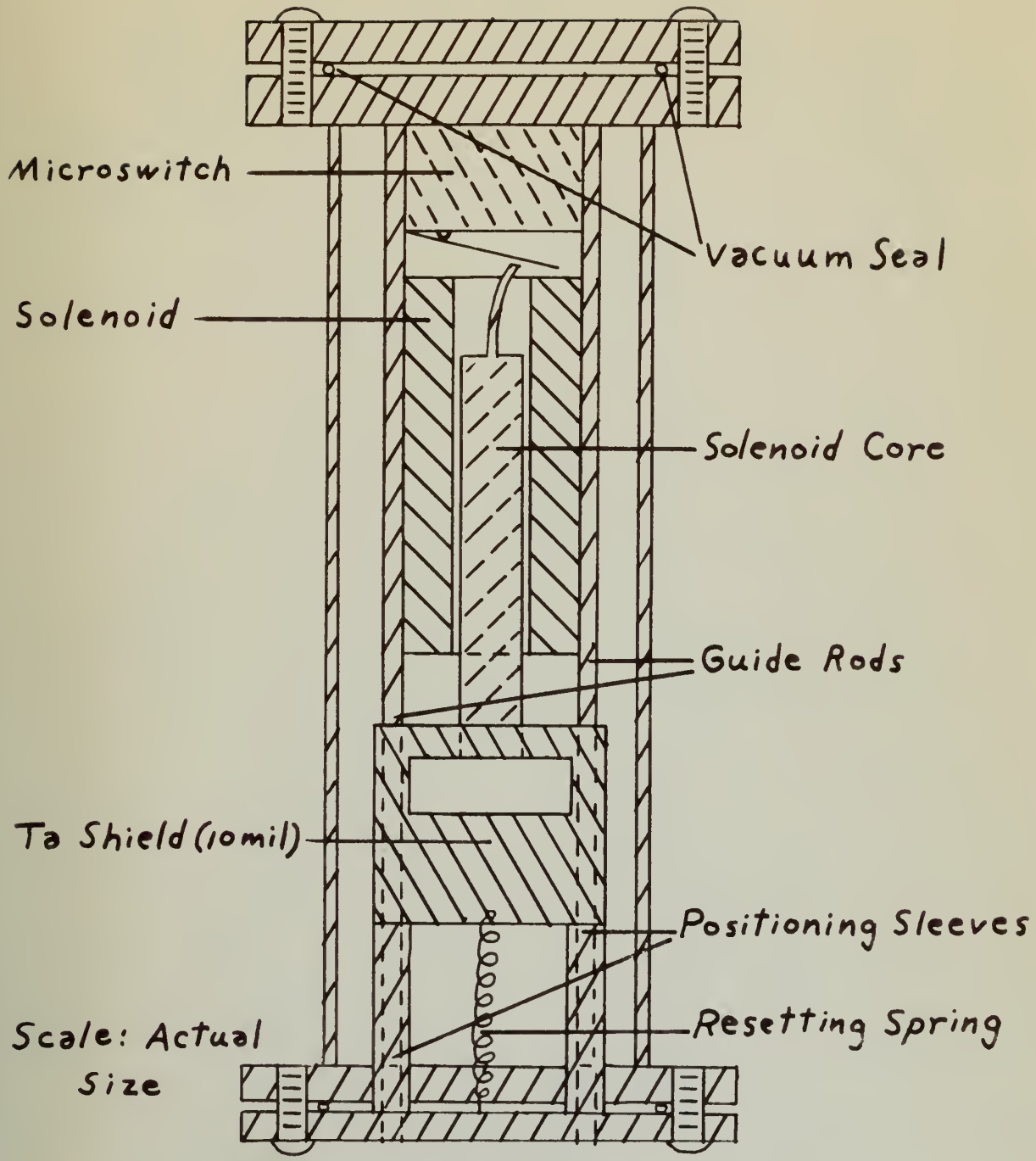

Figure 2. Cross-sectional sketch of shutter.

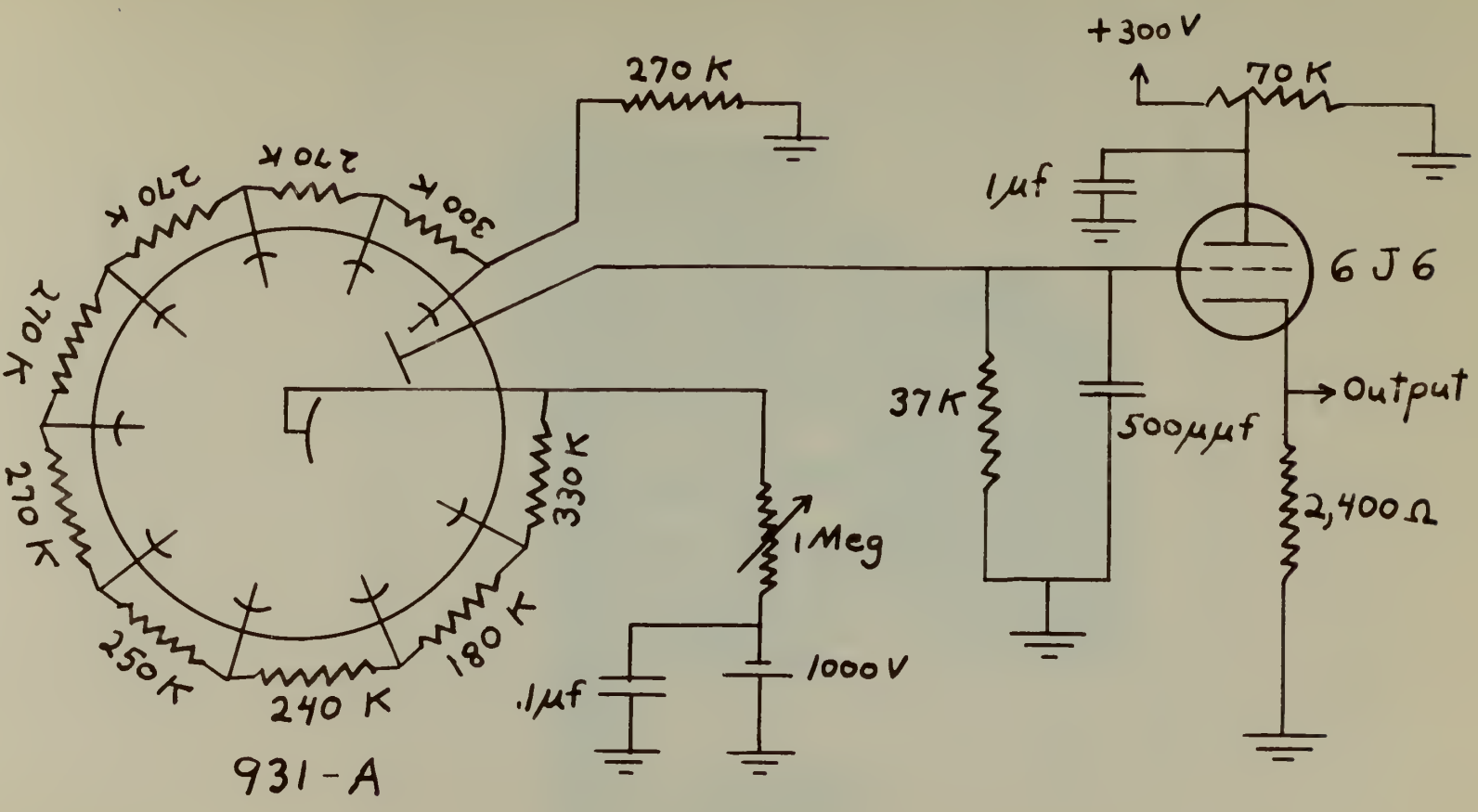

Figure 3. A typical phototube-preamplifier unit.

Figure 4. Photograph of complete phototube - preamplifier unit.

painted black to keep out undesired stray light. A pressure fit for the end of the lucite tubes against the face of the oscilloscope was found to be satisfactory. It should be noted that the variable resistances placed in the phototube circuits permit independent adjustment of the voltage across each phototube (within certain limits). This was found to be necessary since the 931-A tubes used did not have precisely the same characteristics as a function of voltage. It is imperative that the same number of counts be registered by each phototube from the same source of light impulses. By adjusting the voltage properly on each phototube it was possible to attain this desired condition.

Specific Experiments Performed

After this equipment was designed and constructed, it was decided to apply it to the problem of determining the half-lives of the two radioisotopes Al^{25} and Al^{26}, both of which are in the neighborhood of 7 sec. Since there was no sweep time as large as 7 sec. available on the oscilloscope, it became evident that the general method already described would have to be modified slightly for this problem. An auxiliary sweep circuit of the desired length could have been built, but this was decided against in favor of a much simpler idea. Instead of triggering a sweep across

the face of the oscilloscope the electron beam was left focussed on a spot directly under one of the lucite tubes connected to one of the phototubes. The other two phototubes were disconnected completely. Periodically a voltage was applied to the horizontal deflection plates of the oscilloscope to move the spot out from under the lucite tube of the active phototube. This method had the definite advantage that all of the pulses from the Geiger tube were counted while the electron beam spot was focussed under the lucite tube.

The periodic application of a deflecting voltage to the horizontal deflection plates of the oscilloscope was accomplished as follows. A scaler was set to a scale of 64 and then allowed to count the 60 cycle A. C. line frequency. The resulting pulse every 64/60 of a second was fed into an external register. The external register had a sliding metal contact which made contact with the rotating wheel of the mechanical register. However, at definite intervals around the wheel scotch tape was placed so that the electrical contact was broken periodically. In particular the scotch tape was arranged so that the electrical contact was made for 2(64/60) seconds then broken for 3 (64/60) seconds. For convenience hereafter (64/60) sec. will be spoken of as one time unit. With a 22 1/2 volt battery in series with the electrical contact and the horizontal deflection plates of the oscilloscope, the photo-

tube counted for 2 time units, did not count for 1 time units, counted for 2 time units, etc.

A minor difficulty arose when it was found that the pulse coming from the preamplifier following the photo-tube was not large enough to trigger a Tracerlab auto-scaler. It had triggered an Atomic Instrument Co. Model 1030 scaler very nicely, but this scaler was not available for the purpose of counting the phototube pulses. Consequently a two-stage amplifier was designed and constructed to amplify the preamplifier pulse enough so that the auto-scaler would record it. The circuit diagram of this amplifier is given in Figure 5.

Utilizing the modifications mentioned above, the following experiment was performed. A target of isotopically pure* Mg^{24} was bombarded with protons of energy 824 kev. This energy is at a resonance peak for $Mg^{24}(p,\gamma)Al^{25}$. After bombarding for thirty sec. a switch was thrown actuating the solenoid which pulled the shutter up, interrupting the Van de Graaff beam and closing the microswitch. The microswitch's closing actuated a relay which lifted a lead shield from between the target and an active Geiger tube. The Geiger tube then sent pulses to the vertical plates of the

*Obtained from Carbide and Carbon Chemicals Division, Y-12 Area, Oak Ridge National Laboratory, upon allocation from the Atomic Energy Commission.

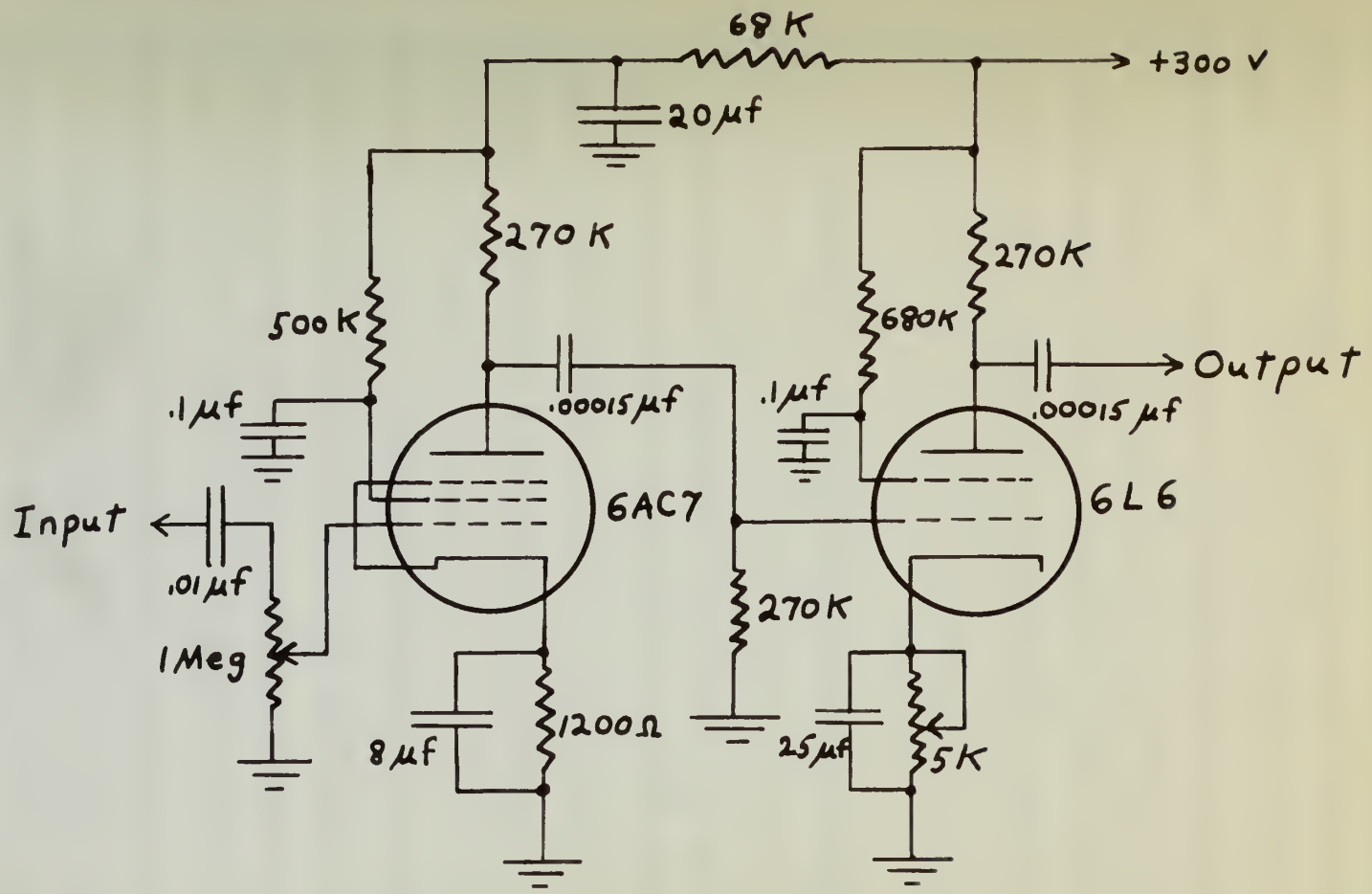

Figure 5. Circuit diagram of amplifier.

oscilloscope and the phototube counted these pulses for 2 time-unit intervals. During the three time-units delay the number of pulses registered in the preceding two time units was recorded. This sequence of events was repeated many times to get sufficient counts to insure a low probable error. The data so obtained are given in Appendix I.

A similar experiment was then performed with a target of Mg^{25}, using protons of energy 565 kev. This energy is at a resonance peak for $Mg^{25}(p,\gamma)Al^{26}$. The data obtained in this experiment are given in Appendix II.

Analysis of Data and Discussion of Results

In the first experiment described the nuclear reaction occurring is known[29] to be $Mg^{24}(p,\gamma)Al^{25}$ so the radioisotope whose activity would be measured is Al^{25}. All of the data obtained for the first time interval (2 time units) were added together as were those for all the other time intervals. This gave ultimately five points on the decay curve of Al^{25}. Figure 6 is a plot on semi-log graph paper of the results obtained from the data listed in Appendix I. The variations plotted are one standard deviation in length. The background counting rate was measured approximately in the middle of each experiment. While taking background data the shutter was closed, with the Van de Graaff beam bombarding it, and the lead shield was raised. Before

started to record the background counts, the induced activity of the target was allowed to decay for one minute.

The value of a standard deviation was obtained in the following manner. Suppose the total number of counts for a given time interval to be N. If there were no background counts, the standard deviation, σ, would be $\sqrt{N}$. The fact that there are background counts must be considered, however. Suppose that the number of background counts in a given time interval is K. Then the standard deviation will be

$$\sigma = \sqrt{N + K} \qquad (11).$$

To actually determine the half-life of Al^{25} from the data in Appendix I a line was fitted to the data using the method of least squares. A short derivation of the theory of this method is given.(18)

Suppose a set of N points, of which only the y coordinate is susceptible to error, when plotted on graph paper suggest a linear relationship with equation

$$y = mx + k \qquad (12).$$

Now, let d_i be the difference between the ordinate of the ith point and the corresponding ordinate of the line. Denote the weight of y_i by w_i. The method of least squares assumes that the best fit will be obtained when the sum of the $w_i(d_i)^2$ is a minimum. This may be expressed as follows:

$$\sum_i w_i d_i^2 = \sum_i w_i \left[y_i - (mx_i + k)\right]^2 \qquad (13)$$

is to be minimized.

Expansion of (13) leads to

$$\Sigma w_i d_i^2 = \Sigma w_i (y_i^2 + m^2 x_i^2 + k^2 - 2my_i x_i - 2ky_i + 2kmx_i) \qquad (14).$$

This may be further simplified to

$$\Sigma w_i d_i^2 = k^2 \Sigma w_i + 2mk \Sigma w_i x_i + m^2 \Sigma w_i x_i^2 - 2k \Sigma w_i y_i - 2m \Sigma w_i x_i y_i + \Sigma w_i y_i^2 \qquad (15).$$

Consider this now as a function of the two variables m and k. Say $\Sigma w_i d_i^2 = f(m,k)$. Now to determine the minima of $f(m,k)$ as functions of m and k respectively one needs only to take the partial derivatives of $f(m,k)$ first with respect to m and then with respect to k and in each case to set the result equal to zero. This gives the following two equations in the two unknowns m and k:

$$\frac{\partial f(m,k)}{\partial m} = 2k \Sigma w_i x_i + 2m \Sigma w_i x_i^2 - 2 \Sigma w_i x_i y_i = 0$$

$$\frac{\partial f(m,k)}{\partial k} = 2k \Sigma w_i + 2m \Sigma w_i x_i - 2 \Sigma w_i y_i = 0 \qquad (16).$$

The solution to these equations may be written down immediately by Cramer's rule and is

$$k = \frac{\begin{vmatrix} \Sigma w_i x_i y_i & \Sigma w_i x_i^2 \\ \Sigma w_i y_i & \Sigma w_i x_i \end{vmatrix}}{D} \; ; \; m = \frac{\begin{vmatrix} \Sigma w_i x_i & \Sigma w_i x_i y_i \\ \Sigma w_i & \Sigma w_i y_i \end{vmatrix}}{D} \qquad (17)$$

where

$$D = \begin{vmatrix} \Sigma w_i x_i & \Sigma w_i x_i^2 \\ \Sigma w_i & \Sigma w_i x_i \end{vmatrix} \qquad (18).$$

$$\begin{vmatrix} m & m^{2} \\ m^{2} & m^{3} \end{vmatrix} \qquad \begin{vmatrix} m & m^{2} \\ m^{2} & m^{3} \end{vmatrix} \qquad (29).$$

$$\begin{vmatrix} m & m^{2} \\ m^{2} & m^{3} \end{vmatrix} \qquad \begin{vmatrix} m & m^{2} \\ m^{2} & m^{3} \end{vmatrix} \qquad (28).$$

$$\frac{\partial^{2}}{\partial x^{2}} \qquad (27).$$

$$\frac{\partial^{2}}{\partial x^{2}} \qquad (26).$$

$$\frac{\partial^{2}}{\partial x^{2}} \qquad (25).$$

$$\frac{\partial^{2}}{\partial x^{2}} \qquad (24).$$

$$\frac{\partial^{2}}{\partial x^{2}} \qquad (23).$$

$$\frac{\partial^{2}}{\partial x^{2}} \qquad (22).$$

$$\frac{\partial^{2}}{\partial x^{2}} \qquad (21).$$

$$\frac{\partial^{2}}{\partial x^{2}} \qquad (20).$$

This gives the solution in general.

Now in this case the actual equation of the radioactive decay being observed is

$$y = ke^{nx} \qquad (19)$$

where x is the time coordinate, y is the counting rate, and n is equal to $(-\lambda)$. To identify this situation with the curve fitting theory just developed one need only take the logarithm of both sides of (19) to get

$$\log_{10} y = \log_{10} k + (n \log_{10} e)x \qquad (20).$$

If one puts $y' = \log_{10} y$, $m' = n \log_{10} e$, and $k' = \log_{10} k$, equation (20) becomes

$$y' = m'x + k' \qquad (21)$$

which is obviously the form already treated.

It has been shown[32] that the proper weighting factor, w_i', to use for the y_i' is $(2.303)^2 y_i^2$. With y_i' substituted for y_i and w_i' substituted for w_i, equation (17) may be used to compute the constants m' and k' in equation (21).

The total number of counts, y_i, obtained in the ith time interval was plotted at the center of the interval. Thus it was the center of each time interval which was denoted as x_i. To be more precise one should use for x_i the quantity $\bar{x}_i$ given by

$$\bar{x}_i = \int_{x_0}^{x_f} x e^{-\lambda x}\, dx \Big/ \int_{x_0}^{x_f} e^{-\lambda x}\, dx \qquad (22)$$

where the limits of integration are from the initial x coordinate to the final x coordinate of the ith interval,

and λ is the decay constant of the radioactive material being measured. Note that an assumed value for λ is necessary to evaluate equation (2). Moreover the fitted value of λ depends on x_i ; hence a method of successive approximations must be used.

The more precise method of determining the x_i's is not necessary if the counting intervals are of constant length, and only the slope of the decay curve is desired. The precise method should be used if one wishes to extrapolate the decay curve back to zero time.

During the actual conduction of the experiment it was observed that following the turning on of the count switch on the scaler, a brief period of time elapsed before any counts were registered. Thus the scaler seemed to have an inherent delay time between the time the count switch was turned on and counts were actually recorded. On the basis of this observation it was necessary to discard the data obtained for the first time interval since actually the scaler was not functioning properly for the full 2 time units. Thus only four points were used to determine the half-lives of the radioisotopes.

Table 1 is a summary of the Al^{25} data needed to compute the half-life. The y_i were determined from the data in Appendix I, and σ was computed by utilizing formula (11).

x_i (time units)	y_i (net counts)	σ_i	y'_i	w'_i ($\times 10^9$)	$x_i w'_i$ ($\times 10^9$)
6	3,192	± 66	3.30406	38.523	195.138
11	1,929	± 56	3.28533	7.170	78.950
16	1,070	± 47	3.02938	1.225	19.600
21	690	± 43	2.83885	.326	6.846
				41.254	300.534

x_i (time units)	y_i (net counts)	$x_i^2 w'_i$ ($\times 10^9$)	$x_i y'_i w'_i$ ($\times 10^9$)	$y'_i w'_i$ ($\times 10^9$)
6	3,192	1,170.828	663.775	113.962
11	1,929	868.538	259.403	23.582
16	1,070	313.600	59.376	3.711
21	690	144.648	19.554	.931
		2,497.614	1,022.108	142.186

Table 1. Summary of Data for Al^{25}.

Using Table 1 and equations (17) one may easily compute n' and k'. The quantity n' is computed to be $-.04515$ (time units)$^{-1}$, while k' comes out to be 3.77559. Now since $n' = n \log_{10} e = .434n$,

$n = -.04515 / .434 = -.1040 = .693 / T.L.$ (23).

From equation (23) the half-life of Al^{25}, T.L., is readily found to be 6.67 time units or 7.11 sec.

It has been shown[32] that an estimate of the standard

error of m' may be obtained from

$$(\text{S.E. in } m')^2 = C_{22}\,\sigma^2_{(ext)} \qquad (24)$$

where

$$C_{22} = \Sigma w_i' / D' \qquad (25)$$

in which D' is the D defined by equation (18) with w_i' substituted for w_i and y_i' for y_i;

and

$$\sigma^2_{(ext)} = \Sigma w_i' d_i^2 / (n-p) \qquad (26)$$

in which n is the number of points to be fitted, and p is the number of adjustable parameters. In this case n-p is 2. Application of equation (24) leads to S.E. in m' = $\pm$.00300 and ultimately the S.E. in the half-life of Al^{25} = $\pm$.20 sec. Thus the half-life of Al^{25} is reported as 7.11 $\pm$ .13 sec., where .13 = (.67)(S.E) = probable error. Figure 6 is a graph of the decay of Al^{25}.

The data listed in Appendix II is summarized in Table 2. Computations precisely similar to those made for Al^{25} were performed for Al^{26}, indicating the half-life of Al^{26} to be 7.61 $\pm$ .21 sec., where the $\pm$ .21 sec. is probable error. Figure 7 is a graph of the decay of Al^{26}.

The values obtained by these experiments for the half-lives of Al^{25} and Al^{26} should now be compared with values reported by other research workers using different methods. Bradner and Gow[19] have reported the half-life of Al^{25} to be 7.3 sec. They bombarded Mg^{25} O with protons from a linear accelerator and found an activity of 7.3 sec. which they

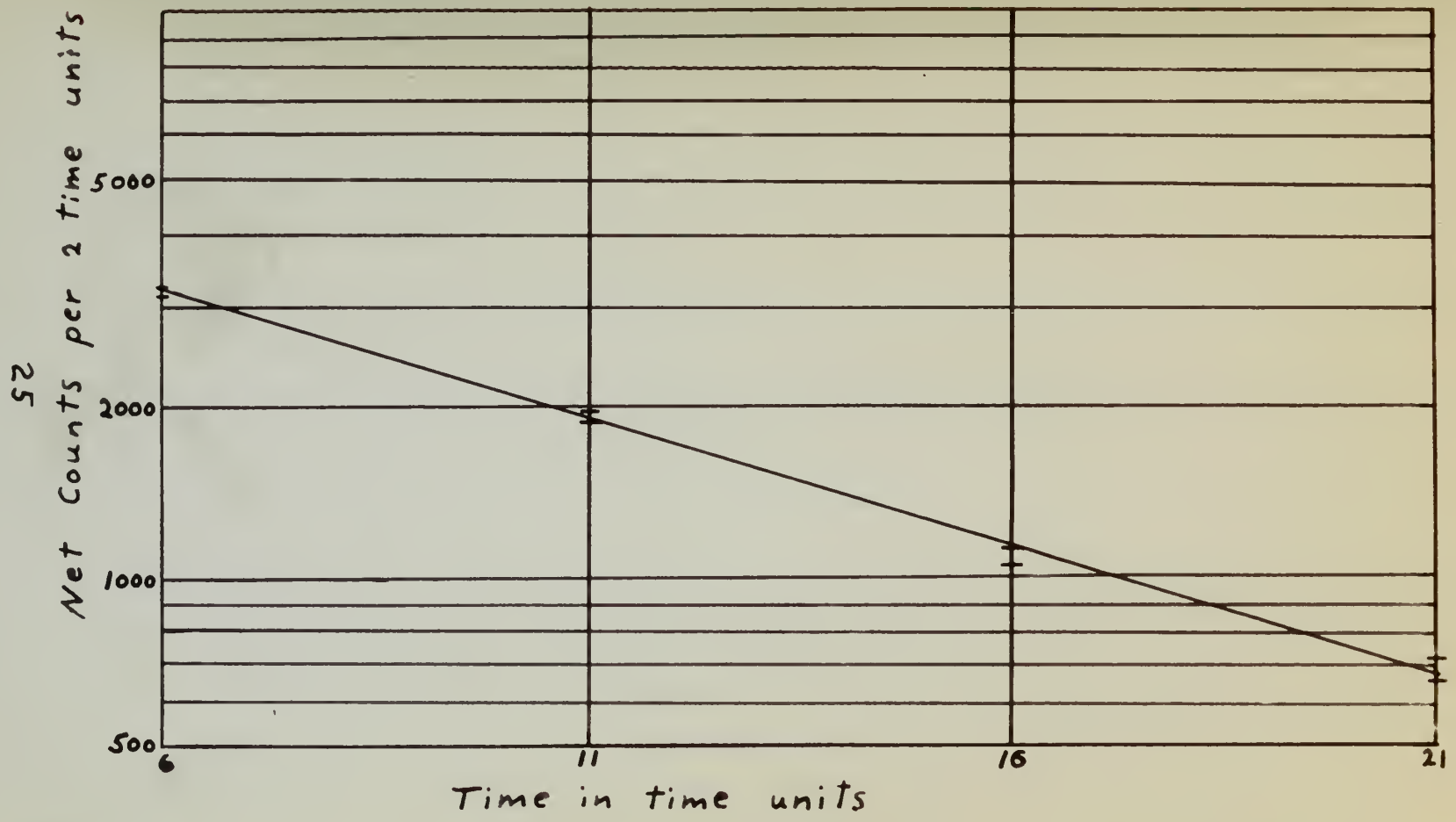

Figure 6. Decay curve of Al^{25}.

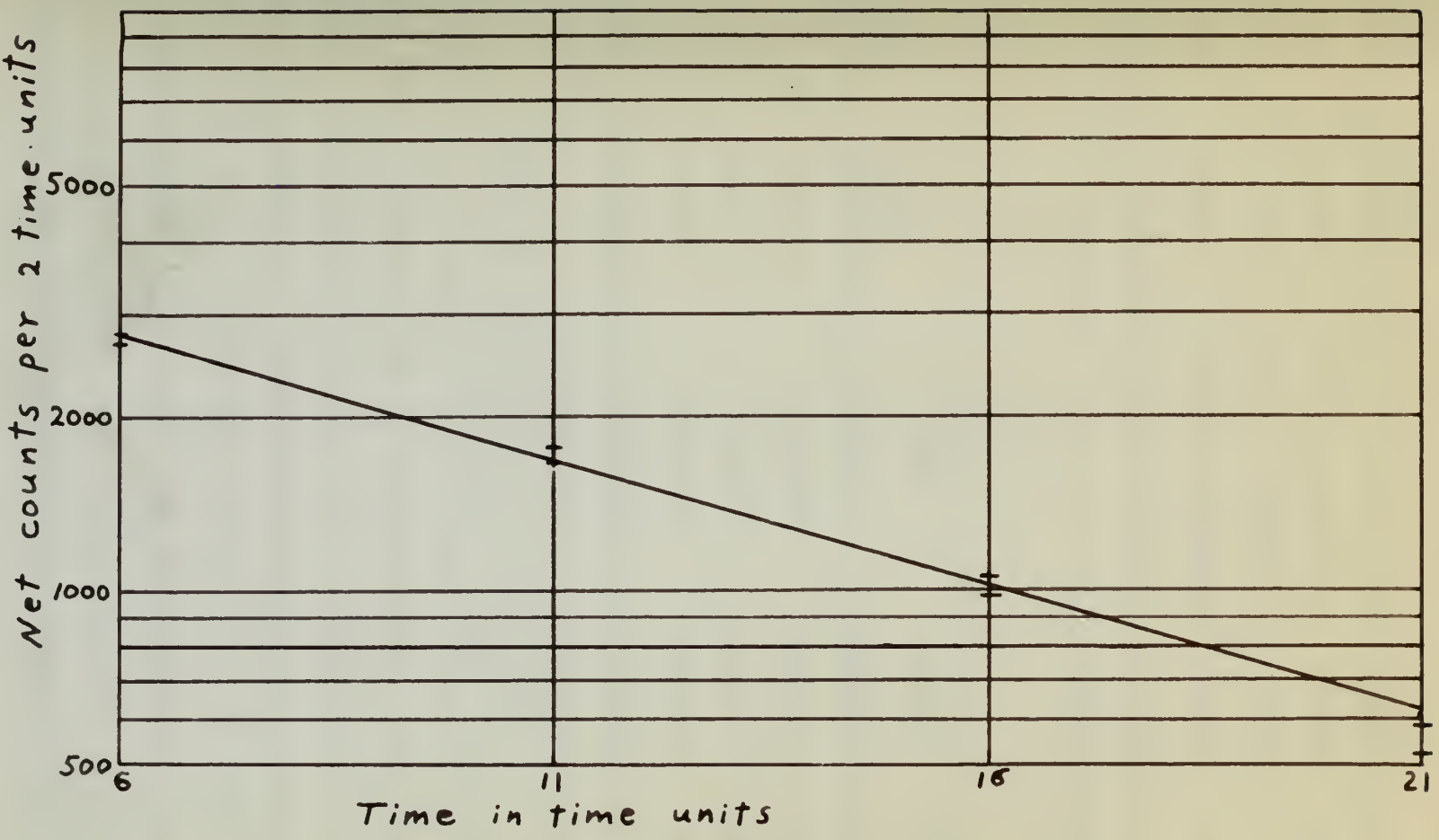

Figure 7. Decay curve of Al^{26}.

x_i (time units)	y_i (net counts)	σ_i	y_i'	w_i' ($\times 10^9$)	$x_i w_i'$ ($\times 10^9$)
6	2,713	± 52	3.43345	19.969	119.814
11	1,713	± 49	3.23378	5.026	55.286
16	1,064	± 41	3.02566	1.024	16.384
21	550	± 35	2.74036	.166	3.486
				26.185	194.970

x_i (time units)	y_i (net counts)	$x_i^2 w_i'$ ($\times 10^9$)	$x_i y_i' w_i'$ ($\times 10^9$)	$y_i' w_i'$ ($\times 10^9$)
6	2,713	718.884	411.379	68.562
11	1,713	608.146	170.702	16.233
16	1,064	367.144	49.259	3.076
21	550	71.206	9.553	.652
		1,662.380	640.893	88.346

Table 2. Summary of Data for Al^{26}.

attributed to the reaction $Mg^{25}(p,n)\ Al^{25}$. It is thus seen that the results of this experiment agree quite closely with those of Bradner and Gow for Al^{25}. Bradner and Gow did not give the details of their experimental arrangement, nor the energy of the bombarding protons, nor probable error in their result. There are very few reports in the literature listing a half-life for Al^{25}. This is undoubtedly due to the fact that only recently have separated isotopes of

ng been available. The common procedure in the past has been to bombard a target of unseparated magnesium with protons. This gives a mixture of the two isotopes Al^{25} and Al^{26}, and since their half-lives are so close together, the decay curve obtained cannot in general be resolved into two components.

Quite in contrast to the case for Al^{25} there are many values reported in the literature for the half-life of Al^{26}. The latest value is 6.3 sec. and was reported by Bradner and Gow in the same article in which they reported on Al^{25}. The reaction they used in this case was $Mg^{26}(p,n)\ Al^{26}$ where again the protons were supplied by a linear accelerator. Other older values[20]-[26] in the literature for the half-life of Al^{26}, however, range from 7.0 sec. to 8.5 sec., with most of the values centering around 7.0 sec. Bradner and Gow have suggested that the 7.0 sec. half-life normally given for Al^{26} is probably a mixture of Al^{25} and Al^{26}. This may be true in some cases but it is difficult to see how it could possibly be true in others. For example, Perlman and Friedlander[20] used the reaction $Al^{27}\ (\gamma,n)\ Al^{26}$ which certainly should have given them almost pure Al^{26}. Any Al^{25} which they would have gotten would have come from the reaction $Al^{27}\ (\gamma,2n)\ Al^{25}$ which has a very small cross section compared to the (γ,n) reaction. The half-life reported by them for Al^{26} was

7.0 sec. Kaffler and Hirzel[21] have also used the Al^{27} (γ,n) Al^{26} reaction; they report the half-life of Al^{26} to be 7.2 sec. In a pioneer experiment in this field Frisch[23], employing the reaction $Na^{23}(\alpha,n)$ Al^{26}, obtained a value of 7 $\pm$ 1 sec. for the half-life of Al^{26}. Huber et al[26], employing the reaction $Al^{27}(\gamma,n)$ Al^{26}, obtained a half-life of 7.2 $\pm$ .5 sec. for Al^{26}. In all cases reported[22],[24],[25] where one would reasonably expect the true activity measured to be a mixture of Al^{25} and Al^{26} only a simple decay curve has been observed. This would tend to indicate that the true half-lives of Al^{25} and Al^{26} are quite close together.

The value found in this experiment for the half-life of Al^{26}, 7.61 $\pm$ .21 sec., does not agree with the 6.3 sec. of Bradner and Gow. No explanation is offered for this discrepancy other than the fact that the value found in this work does agree with that reported by researchers[26] other than Bradner and Gow. One possible explanation is that there are two isomeric states of Al^{26} with quite close half-lives.

Discussion of New Method's Capabilities and Limitations

It is manifestly to be regretted that there was no opportunity to conduct an experiment which would have utilized the new method in its originally described manner.

The main reason for deciding to measure the half-lives of Al^{25} and Al^{26} instead of any other radioisotopes was the fact that this knowledge was desired in collaboration with some work at this university of W. E. Taylor and A. H. Russell on measuring nuclear resonance peaks for the reactions Mg^{24} (p,γ) Al^{25} and Mg^{25} (p,γ) Al^{26}. The author feels certain that if time would have permitted the performance of another experiment, the apparatus described would have worked satisfactorily in its originally described way.

The main advantages of this new method are its simplicity of design, the fact that it utilizes mostly commercially available equipment, and its great flexibility of application over a wide range of half-lives, 1 millisec.to 1 min. For all half-lives longer than 1 sec. the adaptation used in the Al^{25} and Al^{26} experiments is probably more practical than the first-described method.

One of the limitations which should be mentioned includes the large number of runs which would be necessary to insure a small probable error using the first method. Since the half-life would be computed on the basis of only three points, those points would have to be good ones. Also if the method were to be used to measure half-lives shorter than approximately 10^{-3} sec., the Geiger tube should be replaced by a scintillation counter. This would be necessary

since Geiger tubes have dead times on the order of 10^{-4} sec. And too many coincidence counts would be lost. Actually since scintillation counter dead times are of the order of 10^{-8} sec., this method is capable of measuring half-lives on the order of 10^{-7} sec. if an auxiliary fast oscilloscope sweep were constructed, and if an electronic shutter of some type were utilized.

Another minor limitation is the fact that in the modification employed in the current experiments the number of counts obtained at a given voltage setting on the 931-A phototube is very sensitive to the gain control on the amplifier used. This made it necessary to calibrate the phototube part of the apparatus by feeding a known frequency pulse from a pulse generator into the vertical deflection plates of the oscilloscope and adjusting the gain control on the amplifier until the scaler connected to the output of the phototube recorded the correct number of counts. The variable controls were not touched again then during an entire experiment. In reality this is not a serious defect since all that is actually required is that the gain remain constant over the few seconds of each individual run.

There are several radioisotopes whose half-lives might profitably be checked by this new method. Some of these are listed below, together with the reported values of the half-lives.(27),(30),(31)

Isotope	Half-life (sec.)	Reaction	Q (Mev.)
B^{12}	2.7, 2.2 x 10^{-2}	$B^{11}(d,p)B^{12}$	0.4
He^{6}	.85, .97, .8	$Be^{9}(n,\alpha)He^{6}$	-0.8
Li^{8}	.18, .89	$Li^{7}(d,p)Li^{8}$	-0.20
A^{35}	2.2, 1.84, 1.88	$Cl^{35}(p,n)A^{35}$	-3.6
Cl^{33}	2.8, 2.4	$S^{32}(d,n)Cl^{33}$	.65
S^{31}	2.9, 3.2	$P^{31}(p,n)S^{31}$	-3.0
Si^{27}	4.9, 4.5	$Al^{27}(p,n)Si^{27}$	-3.4
P^{16}	7.3, 8	$N^{15}(d,p)P^{16}$	0.5
Mg^{23}	11.6, 11.9, 12	$Na^{23}(p,n)Mg^{23}$	-4.5
N^{12}	12.5 x 10^{-3}	$C^{12}(p,n)N^{12}$	-16.5
Sc^{41}	.87	$Ca^{40}(d,n)Sc^{41}$	.33*
Ca^{39}	1.06	$Ca^{40}(\gamma,n)Ca^{39}$	-14.6*
P^{29}	4.6	$Si^{28}(d,n)P^{29}$	-0.80
K^{37}	1.3	$K^{39}(\gamma,2n)K^{37}$	-24
Ti^{43}	.58	$Ca^{40}(\alpha,n)Ti^{43}$	-2.6*
Cu^{58}	3	$Ni^{58}(p,n)Cu^{58}$	-10.5*
Kr^{97}	1-2	Fission	--
Rb^{90-94}	short	Fission	--
Ca^{41}	?	$Ca^{40}(d,p)Ca^{41}$	6.09

Of course, there are many other short-lived radioisotopes in addition to the ones listed.

*These values were computed using mass values as given by N. Metropolis and G. Reitwiesner, "Table of Atomic Masses", Argonne National Laboratory, 1950.

Another very interesting experiment which could be done would be to measure the half-life of Al^{26} from the reaction $Mg^{25}(p,\gamma)$ Al^{26} for different energies of the bombarding protons. Different values of half-lives at different energies would indicate the existence of isomeric states of Al^{26}. Curran and Strothers[25] performed this experiment with an unseparated Mg target in 1939, and their results suggest the possibility that indeed different half-lives might be found at different bombarding proton energies.

Summary

A new method of measuring short half-lives in the range from 1 millisec. to 1 min. has been described. This method has then been applied to the problems of determining the half-lives of Al^{25} and Al^{26} with the following results: Al^{25}-7.11 ± .13 sec. ; Al^{26}-7.61 ± .21 sec. Some capabilities and limitations of the method have been discussed and future applications have been suggested.

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Appendix I

Data for Mg^{24} (p,γ) Al^{25} experiment.

Background: 1084 counts in 467 sec.

All time intervals 2 time units or 2(64/60) sec. long.

The numbers recorded are 2 times the actual cumulated counts.

1st	2nd	3rd	4th	5th
92	162	188	226	248
72	86	120	132	132
112	194	252	302	320
94	122	176	210	280
74	110	140	174	190
84	148	216	228	242
54	84	130	158	170
64	108	152	152	178
36	60	146	144	148
136	214	286	296	358
168	172	206	260	276
176	292	334	356	366
96	160	218	274	314
78	136	154	174	190
64	92	118	118	136
90	102	230	266	266
154	248	308	450	470
134	172	196	222	256
172	234	248	354	406
60	204	240	290	310
16	22	22	38	42
92	116	134	154	186
60	110	124	140	174
68	122	182	206	236
116	204	260	290	304
36	86	110	134	154
126	208	242	254	278
42	66	78	102	114
110	214	282	332	362
230	394	500	524	550
186	336	386	426	448
56	76	110	110	120
38	74	76	90	96

1st	2nd	3rd	4th	5th
46	74	96	104	112
60	66	102	102	116
12	22	24	32	32
152	256	304	348	358
126	196	230	244	270
146	174	196	240	260
12	22	22	32	44
30	38	42	46	64
62	116	134	160	184
46	80	140	148	150
108	132	166	212	220
78	124	150	160	160
38	62	94	154	166
72	88	144	172	208
134	178	218	260	264
120	232	282	312	350
124	166	194	230	212
58	96	118	144	168
60	102	132	132	140
54	146	184	210	230
30	74	112	128	128
82	96	110	114	128
116	212	274	310	320
76	136	160	166	172
118	160	192	198	212
76	136	168	198	206
88	144	168	192	216
112	112	140	180	180
114	236	298	322	374
136	212	304	360	370
196	390	456	512	532
76	132	148	170	198
92	140	192	208	236

[illegible]	40	42	50	84
[illegible]	96	110	132	162
90	54	90	116	148
[illegible]	112	172	202	226
108	132	192	228	240
96	170	214	252	264
76	104	160	190	216
[illegible]	114	170	230	266
90	90	136	158	184
86	130	172	178	216
[illegible]	124	170	186	212
[illegible]	40	66	66	98
58	66	130	140	156
72	126	146	150	160
108	150	196	210	216
126	170	236	270	294
108	208	262	282	290
108	140	162	174	212
36	74	84	94	96
16	38	56	60	60
52	100	126	172	172
58	150	156	184	216
32	56	84	106	124
90	126	172	202	220
54	72	72	72	78
76	132	150	162	196
52	96	136	144	148
100	166	224	230	260
92	104	156	200	256
94	142	162	182	200

72	118	154	220	236
58	156	162	186	212
60	100	234	244	252
104	168	200	210	236
80	116	240	258	284
100	168	234	296	314
90	178	272	304	310
76	202	222	268	278
106	174	224	250	260
44	128	172	178	194
92	200	250	270	292
120	186	220	242	246
82	136	160	186	198
166	236	314	368	412
96	148	210	236	236
132	170	206	260	276
96	204	250	314	376
46	118	148	186	186
126	168	252	308	384
114	226	246	292	292
72	138	206	224	256
120	238	298	340	372
76	106	142	214	272
132	192	264	282	302
140	244	282	318	368
86	236	278	300	336
152	196	224	236	242
156	238	336	348	396
138	216	262	320	340
192	276	358	362	416

	1st	2nd	3rd	4th	5th
Sums	11,308	16,872	23,910	27,180	29,790
÷ 2	5,654	9,436	11,955	13,615	14,895
Differences	5,654	3,782	2,519	1,660	1,280
- S. D.	5,064	3,192	1,929	1,070	690

Appendix II

Data for Mg^{25} (p,γ) Al^{26} experiment.

Background: 75% counts in 1,203 sec.

All time intervals 2 time units or 4(54/10) sec. long.

The numbers recorded are 2 times the actual cumulated counts.

1st	2nd	3rd	4th	5th
24	36	[illegible]	90	96
16	24	[illegible]	32	56
4	12	30	40	50
24	30	36	50	42
18	34	[illegible]	70	74
26	42	52	56	64
34	60	84	96	108
18	42	60	68	72
34	48	56	58	60
34	60	78	84	88
12	26	36	38	42
[illegible]	76	[illegible]	100	116
40	64	72	92	92
32	58	76	92	96
28	50	70	76	80
30	52	76	80	88
40	76	106	122	134
28	66	70	78	90
24	38	[illegible]	66	70
28	54	68	90	102
50	68	78	[illegible]	94
46	74	96	[illegible]	120
30	48	58	60	66
14	22	44	28	30
6	8	14	18	[illegible]
10	12	18	20	[illegible]
24	50	78	[illegible]	88
44	80	[illegible]	98	102
34	[illegible]	78	90	[illegible]
58	78	[illegible]	112	112
52	74	106	118	126
30	60	72	92	80
32	60	82	98	[illegible]
[illegible]	50	78	86	90
62	80	106	112	112

1st	2nd	3rd	4th	5th
18	42	26	38	34
6	20	24	34	36
18	50	66	72	74
28	54	58	62	66
28	56	72	82	90
24	38	[illegible]	56	64
56	72	82[illegible]	[illegible]	100
20	50	52	60	68
30	56	72	[illegible]	96
56	70	90	102	106
24	[illegible]	[illegible]	86	94
18	[illegible]	52	58	64
22	48	72	82	92
30	56	66	68	72
24	36	50	[illegible]	70
34	70	106	122	128
44	76	94	107	108
18	[illegible]	38	66	76
18	28	32	[illegible]	58
44	64	78	[illegible]	98
36	58	68	80	90
24	36	50	58	[illegible]
12	16	24	34	36
[illegible]	[illegible]	30	[illegible]	[illegible]
10	16	24	24	28
30	[illegible]	34	58	64
46	76	100	126	134
32	50	70	72	[illegible]
32	40	64	96	96
46	72	90	100	104
32	42	52	58	70
58	88	106	124	130
[illegible]	82	120	132	126
50	68	76	[illegible]	86
16	44	50	66	78

[illegible]	36	62	42	62	24	62	60	80	74
17	56	[illegible]	[illegible]	[illegible]	14	74	54	84	80
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	24	[illegible]	42	56	56
32	18	32	[illegible]	[illegible]	14	22	14	36	62
[illegible]	[illegible]	72	[illegible]	[illegible]	12	46	50	64	68
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	28	40	46	54	56
[illegible]	[illegible]	36	[illegible]	[illegible]	[illegible]	60	62	50	50
[illegible]	[illegible]	12	[illegible]	[illegible]	14	16	40	[illegible]	42
[illegible]	[illegible]	[illegible]	30	32	[illegible]	12	36	38	36
[illegible]	[illegible]	82	100	104	16	54	52	56	58
[illegible]	74	80	98	98	18	60	56	72	84
[illegible]	78	112	112	130	50	82	96	100	112
[illegible]	80	100	122	128	42	84	106	112	122
[illegible]	[illegible]	[illegible]	96	110	46	66	82	112	116
[illegible]	76	104	114	108	56	80	94	102	108
[illegible]	56	66	90	88	42	78	84	92	112
[illegible]	78	92	96	112	24	50	68	76	90
[illegible]	[illegible]	56	74	88	28	48	52	82	68
[illegible]	[illegible]	104	110	130	66	102	116	132	138
[illegible]	[illegible]	80	90	96	32	64	86	96	104
[illegible]	72	100	108	114	60	94	102	100	102
[illegible]	[illegible]	120	120	122	56	92	100	114	120
[illegible]	76	96	110	120	56	96	124	124	124
[illegible]	74	88	94	106	38	64	64	90	100
[illegible]	[illegible]	92	100	112	64	96	124	136	142
[illegible]	78	108	124	128	30	48	68	80	82
53	84	110	116	116	34	68	68	100	112
[illegible]	[illegible]	[illegible]	76	82	24	34	68	[illegible]	54
[illegible]	86	74	78	62	30	52	66	80	80
[illegible]	[illegible]	78	84	88	44	74	88	96	106
[illegible]	[illegible]	42	56	96	28	42	82	100	100
[illegible]	90	60	82	96	26	42	74	82	80
[illegible]	80	76	88	96	54	76	102	114	118
[illegible]	[illegible]	98	108	108	54	42	72	88	66
[illegible]	[illegible]	62	96	78	32	46	62	76	78
[illegible]	[illegible]	80	92	114	44	46	84	92	94
[illegible]	52	78	92	96	24	32	52	60	70
[illegible]	[illegible]	110	108	108	18	20	40	56	54
[illegible]	[illegible]	96	114	114	6	20	36	44	48
62	[illegible]	108	120	120	40	64	82	90	104
36	[illegible]	84	72	78	34	56	64	76	86
34	[illegible]	78	80	96	12	56	68	72	80
74	[illegible]	108	112	142	20	48	62	72	76
36	[illegible]	72	78	92	28	32	50	70	60
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	30	44	52	72	76
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	72	[illegible]	[illegible]	80
[illegible]	[illegible]	84	54	64	[illegible]	52	56	64	74
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	24	[illegible]	[illegible]	48
[illegible]	[illegible]	44	56	[illegible]	31	56	[illegible]	[illegible]	92
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	44	[illegible]	[illegible]	84
20	36	84	88	58	14	32	40	44	32

| | | |

32	44	62	64	78
34	45	70	76	82
36	47	72	78	88
38	50	75	86	90
40	52	78	88	94
42	54	79	90	98
44	56	82	92	100
46	58	84	94	102
48	60	86	96	104
50	62	88	98	106

32	44	62	64	78
34	46	64	66	80
36	48	66	68	82
38	50	68	70	84
40	52	70	72	86
42	54	72	74	88
44	56	74	76	90
46	58	76	78	92
48	60	78	80	94
50	62	80	82	96

32	44	62	64	78
34	46	64	66	80
36	48	66	68	82
38	50	68	70	84
40	52	70	72	86
42	54	72	74	88
44	56	74	76	90
46	58	76	78	92
48	60	78	80	94
50	62	80	82	96

	1st	2nd	3rd	4th	5th
Sum	9,114	15,260	19,380	31,102	33,702
½	4,557	7,630	9,690	14,051	11,251
Differences	4,567	1,063	2,063	1,358	990
- A. D.	4,217	2,713	1,713	1,008	590

Date Due
